§10 Analyzing Arguments

There are two argument types we consider.

Definition: Inductive Argument - an argument that supports a general conclusion using specific examples, presented as premises.

Ex: Premise: $-2 \cdot -3 = 6$

Premise: $-8 \cdot -1 = 8$

Premise: $-100 \cdot -2 = 200$

Premise: $-.5 \cdot -2 = 1$

Conclusion: A negative times a negative is a positive.

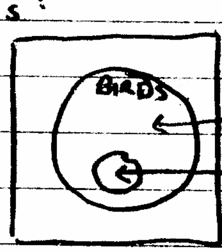
Ex: Premise: Swans are birds and they can fly.

Premise: Geese are birds and they can fly.

Premise: Sparrows are birds and they can fly.

Conclusion: All birds can fly.

Graphically, the second example looks like this:



set of all birds

{swans, geese, sparrows}

IE, the set of birds the argument presents in its premises.

The problem is that set is only a subset of the set of birds. What if we missed a member of the birds set that cannot fly? Well, then the argument is wrong.

NOTE: This presents a vital thing... AN INDUCTIVE ARGUMENT CANNOT PROVE ITS CONCLUSION. This is because an inductive argument looks only at a SUBSET of the set representing its conclusion. Hence, there is a possibility that one member of the larger set does NOT behave as the argument suggests. Since this doubt always exists for an inductive argument, it can't prove anything!

Strength of an inductive argument:

We can rate an inductive argument as

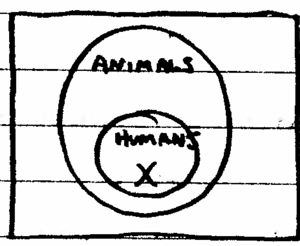
* weak (few premises or an obvious counter-example)
* Somewhat strong (presents a good case for its conclusion in its premises.)
* Strong (presents very strong evidence to support its conclusion.) Few inductive arguments are in this category.

Definition: deductive argument - an argument that supports a conclusion with more general premises.

NOTE: It IS possible for a deductive argument to prove its conclusion if its premises are true and its conclusion follows from them.

Ex: All humans are animals. ← Premise
Doug is a human. ← Premise
Doug is an animal. Conclusion

We represent this with a venn diagram:
The first premise is the "All p are q" form, so we use the subset:



The "X" comes from the second premise - Doug is a human. Obviously, since the "X" is inside both the human and animal circle, this proves the conclusion that Doug is an animal.

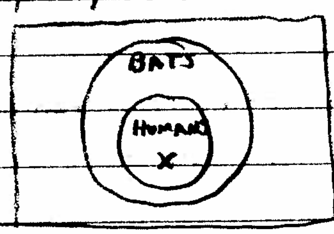
Testing Deductive Arguments:

Definition: A deductive argument is <u>valid</u> if its conclusion follows necessarily from its premises. IE, just assume the premises are true. Does that force the conclusion to be true? If so, the deductive argument is <u>VALID</u>

Definition: A deductive argument is <u>sound</u> if it is valid AND all of its premises are true.

Ex: All humans are bats. ← Premise
Doug is a human. ← Premise
Doug is a bat Conclusion

Following the same procedure as in the previous example, we have the venn diagram:



This argument <u>is VALID</u> since the second premise places the X inside the human circle and, thus, inside the bats circle. So, assuming all humans are bats and Doug is a human, it MUST be true that Doug is a bat, since he is human, and all humans are bats.

This is NOT a sound argument however, since it is clear the first premise is false.

This was an example of a deductive argument that is

Valid ✓

Sound X

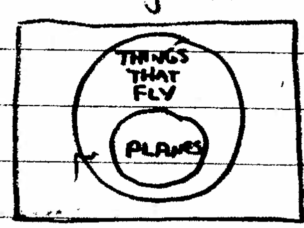
Here is an example of a deductive argument that is Invalid (and thus not sound)

Ex: Premise: All planes fly.

Premise: Sparrows are not planes.

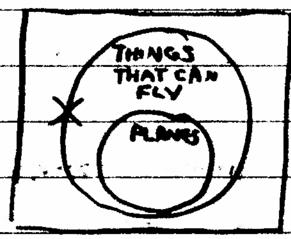
Conclusion: Sparrows can't fly.

Creating the venn diagram from premise one gives:



Since we said "all planes fly", planes must be a subset of "things that can fly." That explains the two circles. What is the rectangle? Its the universal set. Let's say it contains everything on Earth.

The Venn diagram we drew comes from the first proposition. We now must consider the second proposition. This involves, as in the previous two examples, placing an "X" in the region specified by the proposition. It says that Sparrows are NOT planes. So sparrows are NOT in the PLANE circle. So the "X" goes outside of that circle. But which region? There are two possible regions to choose from. We don't know which to choose, so we place the "X" on the border of the two possible regions.



This tells us that sparrows are not planes. Since the "X" represents sparrows, and the "X" is on the border, we don't know from the argument whether sparrows can fly.

Thus, this example IS NOT VALID since the premises don't "prove" the conclusion. If they did, the "X" would be outside of the "Things that can fly" circle. Since the definition of Soundness requires Validity to hold, the argument is not sound.

Valid? X

Sound? X

Example where the argument is invalid, but the Conclusion is clearly valid.

Ex: Premise: 1+1=2.

Premise: Soccer is a sport.

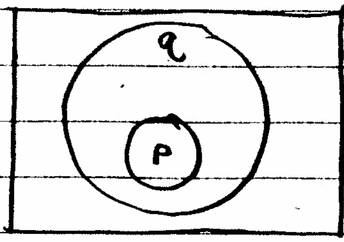
Conclusion: The world is not flat.

The conclusion is obviously true, but the argument is absurd. This argument is not Valid, and it is also not Sound. This makes an important point:

YOU MUST ALWAYS DRAW THE VENN DIAGRAM AND "PLANT THE X" TO DETERMINE SOUNDNESS AND VALIDITY. THE CONCLUSION BEING TRUE IN NO WAY GIVES YOU A SHORTCUT TO THE ANSWER! NOR DOES THE CONCLUSION BEING FALSE GIVE YOU ANY SHORTCUTS. (See the Second example in §1C Notes)

Conditional deductive arguments

The venn diagram for the If p, then q conditional is "p is a subset of q"

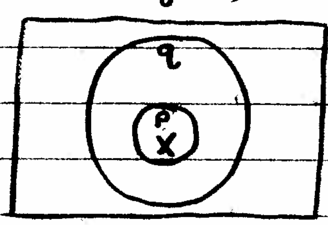


Using this Venn diagram and the same logic as in the previous examples, we can analyze these arguments.

① Affirming the Hypothesis

Structure: If p, then q
<u>given p is true.</u>
q is then true

This is ALWAYS VALID. For p and q, your venn ~~diagram~~ will look like this

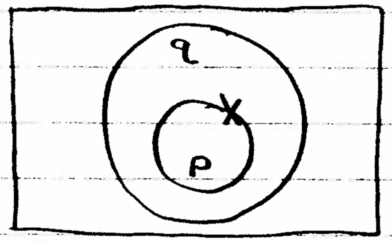


② Affirming the Conclusion

structure: If p, then q
<u>given q is true</u>
p is true

This is INVALID. Why? Because when you draw the venn diagram and "plant the x" you find the x is on a boarder.

The venn diagram is:



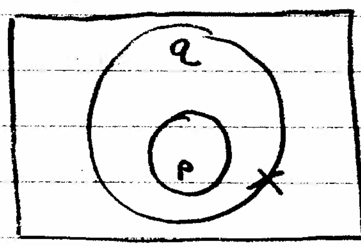
③ Denying the Hypothosis

Structure: If p, then q

given p is NOT true

q is NOT true.

This is an INVALID argument. It's Venn diagram is:



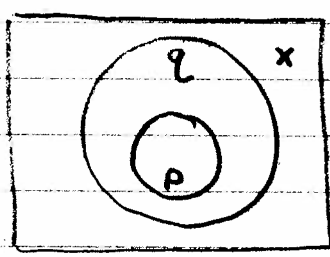
④ Denying the conclusion

Structure: If p, then q

q is NOT true is given.

p is NOT true

This is VALID. After all, if q is false, p cannot be true either since p is a subset (inside of) q. The Venn diagram:



Since the "x" is not on a border, the argument is valid.

Using induction to test mathematical rules:

Ex: Use induction to determine if the following is true:

a) $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$, where a and b are whole numbers.

Since we want to use induction, we test with examples.

Does $\sqrt{0+0} = \sqrt{0} + \sqrt{0}$?

Yes.

Does $\sqrt{1+0} = \sqrt{1} + \sqrt{0}$?

Yes

Does $\sqrt{1+3} = \sqrt{1} + \sqrt{3}$?

No

This shows that $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$ in all cases.

b) Does $\frac{2}{3} + \frac{a}{b} = \frac{2a}{3b}$ when a and b are counting numbers?

Using induction:

Does $\frac{2}{3} + \frac{5}{3} = \frac{2 \cdot 5}{3 \cdot 3}$?

No, since $\frac{7}{3} \neq \frac{10}{9}$

Thus, it is clear this is NOT true. IE, inductively we see that the conclusion $\frac{2}{3} + \frac{a}{b} = \frac{2a}{3b}$ is false.